Solutions de quelques exercices de recherche de primitives.

1) $$\int \frac{2x^2+5x-3}{x}\,dx = \int\left(2x+5-\frac{3}{x}\right)dx = x^2+5x-3.\ln|x|+c.$$

2) $$\int 3x.\sqrt{4-x^2}\,dx = \int 3x.(4-x^2)^{\frac{1}{2}}\,dx = \frac{3}{-2}\int -2x\,(4-x^2)^{\frac{1}{2}}\,dx$$

$$= -\frac{3}{2}.\frac{(4-x^2)^{\frac{3}{2}}}{\frac{3}{2}}+c = -(4-x^2)^{\frac{3}{2}}+c$$

Autre façon : $u = 4-x^2 \rightarrow du = -2x\,dx$

$$\int 3.\sqrt{u}\,\frac{du}{-2} = -\frac{3}{2}\,\frac{u^{3/2}}{3/2}+c = -u^{3/2}+c = -(4-x^2)^{\frac{3}{2}}+c.$$

3) $$\int x^2.\sin x\,dx = -x^2.\cos x + 2\underbrace{\int x.\cos x\,dx}_{J}$$

$$\begin{cases} u = x^2 \rightarrow u' = 2x \\ v' = \sin x \rightarrow v = -\cos x \end{cases}$$

Calcul de J : $$\begin{cases} u = x \rightarrow u' = 1 \\ v' = \cos x \rightarrow v = \sin x \end{cases}$$

$$J = x.\sin x - \int \sin x\,dx$$

$$\int x^2.\sin x\,dx = -x^2.\cos x + 2x\sin x - 2\int \sin x\,dx$$

$$= -x^2.\cos x + 2x.\sin x + 2.\cos x + c.$$

4) $$\int \frac{1}{\sqrt{1-25x^2}}\,dx = \int \frac{1}{\sqrt{1-u^2}}.\frac{du}{5} = \frac{1}{5}\arcsin u + c$$

($u = 5x \rightarrow du = 5\,dx$) $$= \frac{1}{5}\arcsin(5x)+c$$

5) $$\int \frac{8x^2}{x^3-1}\,dx = \frac{8}{3}\int \frac{3x^2}{x^3-1}\,dx = \frac{8}{3}.\ln|x^3-1|+c$$

(on peut aussi poser $u = x^3-1$)

6) $$\int \frac{8x^2}{(x^3-1)^2}\,dx = \int 8x^2.(x^3-1)^{-2}\,dx = \frac{8}{3}\int 3x^2.(x^3-1)^{-2}\,dx = \frac{8}{3}.\frac{(x^3-1)^{-1}}{-1}+c$$

$$= \frac{-8}{3.(x^3-1)}+c$$

(on peut aussi poser $u = x^3-1$
$\rightarrow du = 3x^2\,dx$ d'où : $\int \frac{8}{u^2}.\frac{du}{3} = \dots$)